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Fractals and picturesque composition

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Abstract. In this paper fractals are identified in classical mouldings by means of an algorithm for drawing Levy staircases. It is argued that traditional rules of composition favour fractal forms such as these, and advice on composition, taken from Ruskin, is examined to support this view. Because fractals are ubiquitous in nature their use in design offers a way to appear natural. The lack of fractals in modern architecture may therefore be connected to a lack of interest in picturesque composition.

Introduction: fractals and buildings

A building is fractal if it repeats and multiplies a form, such as a pointed arch, over several orders of size. Gothic buildings aside, not many buildings actually do this, but a good example of one that does would be the memorial at Thiepval (figure 1), which repeats an arch at four sizes. This fractal has the macabre purpose of providing a large surface area for recording the names of those missing in the Battle of the Somme. Modern examples are harder to find—the well-known flats in west Amsterdam by architects MVRDV (figure 2) repeats a box shape over three stages, although the opportunity to carry the progression below the size of a balcony is not taken. Both these examples are artfully composed and somewhat picturesque.

Buildings can also be fractal by having the number of elements falling within a particular size range obey a power law. This method treats a building like a landscape where elements of a particular size are counted without wondering why they are the shape they are. This is an easier way for a building to be fractal than if all its parts are made to have something in common. Most old buildings are fractal in this sense because they have a lot of small-scale detail; in short, because they have mouldings.

Figure 1. Thiepval memorial (by Lutyens in 1925).

Figure 2. Amsterdam housing (by MVRDV in 1995).

Most modern buildings are not fractal even in this easier sense because they use large blank surfaces, avoid decoration, and use simple forms, such as a single block, rather than one broken into subsidiary volumes. These reasons why modern architecture is not fractal are the same reasons that are often given to explain why it can appear oppressive and unnatural.

In what follows, it will be argued that being fractal and being picturesque go together. This applies to buildings and to fractals themselves, even those drawn by simple geometrical rules such as the Koch curve (see figure 8). The Koch curve, which looks a little like the silhouette of a tree, is picturesque because of its irregular rhythms and because of the proportions between its parts, as will be demonstrated later. I will also consider rules of composition in relation to fractal architecture. This could be done by looking at the picturesque aspects of gothic architecture; however, rather than examining a particular fractal building, it will be more useful to examine classical mouldings, a fractal architectural form which is common to many buildings.

An algorithm to generate mouldings

How does a Doric entablature, built up out of just a few steps and curves, manage to appear so graceful and natural? Figure 3 is drawn according to the algorithm in Normand (1931), which uses only dimensions that are fractions of the diameter of the column at its base, the smallest subdivision used being a twenty-fourth part of that distance. The lack of exact repetition in the entablature suggests that the algorithm is minimal but parts of it do seem to resemble the whole. This is illustrated in figure 4

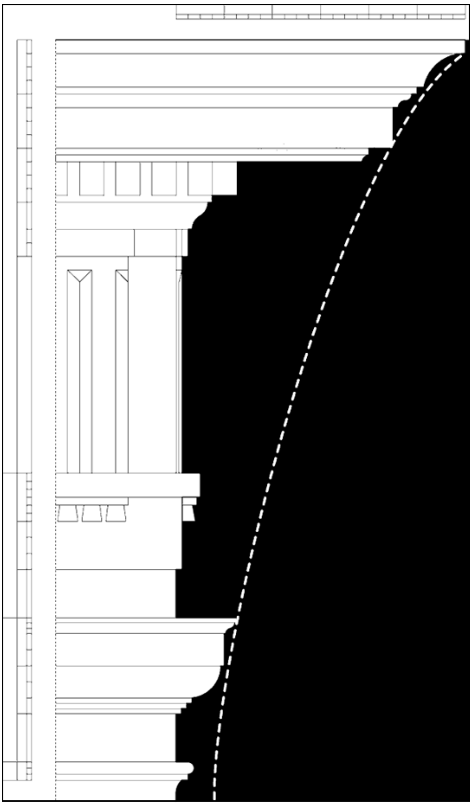

Figure 3. A denticular Doric entablature.

Figure 4. Scaling over four stages in a Doric cornice.

where the Doric cornice may be seen roughly to repeat itself inwardly over four stages as the parts in the boxes are stretched to the proportions of the original. Notice that the whole entablature divides naturally into three parts, namely the cornice, frieze, and architrave, all of which divide internally into three parts; in fact the whole thing seems to be constructed in threes. This threeness of classical ornament takes on an added significance when placed next to the well-known fractal 'the Devil's staircase' (Mandelbrot, 1983, page 80), the resemblance of which to a cornice is most striking (figure 5). This lack of exact periodicity combined with repetition at smaller scales suggests that classical mouldings are fractals. To test this idea, slightly irregular versions of the Devil's staircase were drawn with different fractal dimensions to see how closely they could be made to resemble the profile of mouldings. These shapes, known as Levy staircases, (Mandelbrot, 1983, pages 23–24, page 131), were constructed by making all the horizontal steps, Δx, the same size whilst allowing the vertical steps, Δy, to vary randomly subject to the condition that the probability, p, that Δy is greater than a particular size is hyperbolic, that is to say $\mathrm{p}(\Delta y > u) = u^{-D}$. Finally, the steps were rescaled so that $\sum \Delta x = \sum \Delta y$ to make the staircase fit diagonally inside a square. The results of taking 500 equal horizontal steps with values of D varying from 0.1 to 0.9 are shown in figure 6 (see over). The staircases shown there are unselected, being the first forty five produced by the program. The hyperbolic distribution of step sizes allows a reasonable probability of large jumps unlike, for example, a Gaussian distribution where the probability of step sizes far away from an expected value becomes vanishingly small. Although large vertical steps are less likely than small ones, and the higher the value of D the less frequent they become, the possibility that they may occur remains and it is this that makes the set of vertical steps a self-similar fractal of dimension D. It is perhaps not obvious that all the staircases in figure 6 really do ascend at each of the 500 horizontal increments because when by chance a very large jump occurs the detail of the smaller steps is lost when the staircase is rescaled to fit inside the square. This happens most commonly when D is low. If, however, one enlarged part of the staircase to the size of original in the manner of figure 4 then this detail would reappear.

Many of the staircases are plausible approximations to mouldings. The best are those with a dimension of about 0.6 in which clusters of small steps resemble the

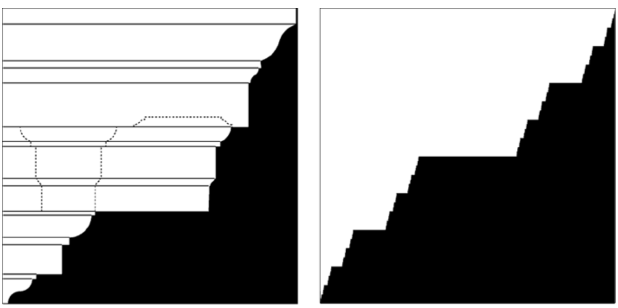

Figure 5. A composite cornice (left) and the Devil's staircase compared.

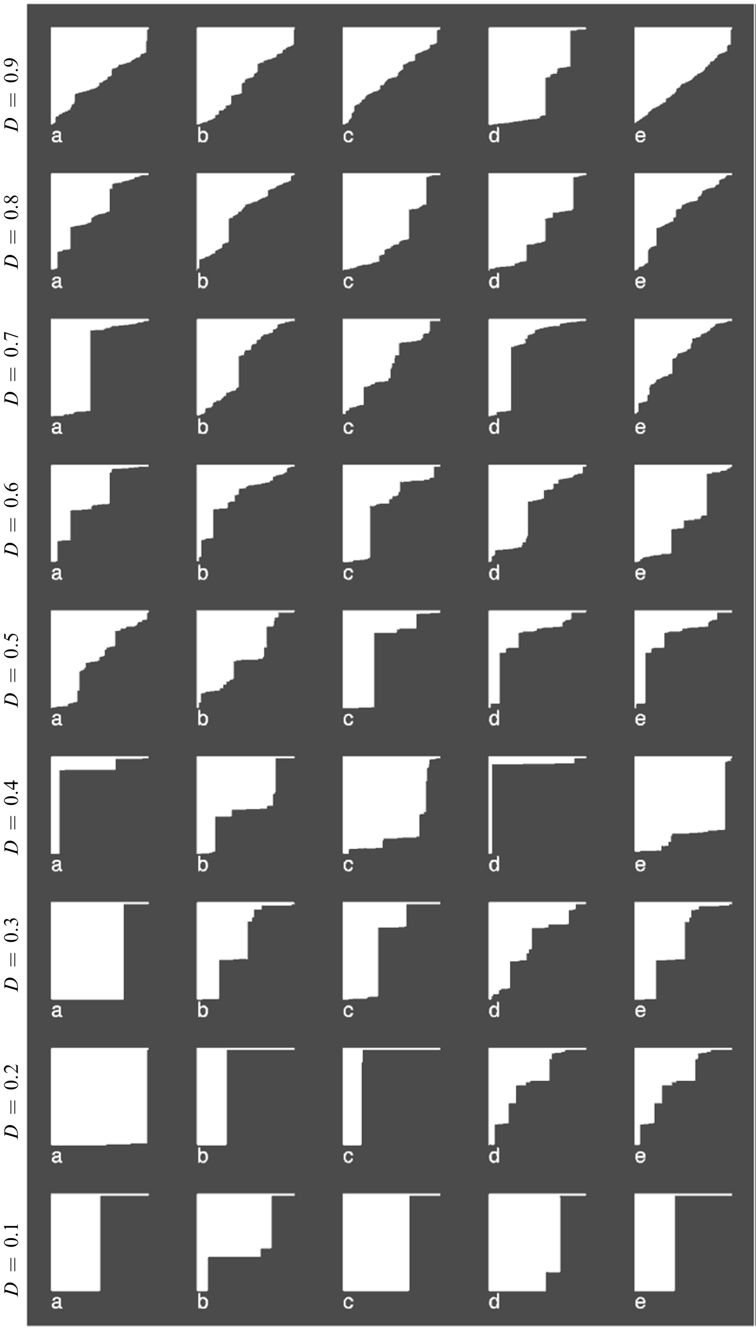

Figure 6. Unselected Levy staircases of dimensions $D = 0.1$ to 0.9.

curved parts of mouldings and the balance of large and small steps seems about right. To test how important this element of choice was the program was run about thirty times with the dimension set to 0.6 and the most appealing three selected—these are illustrated in figure 7. If scaling was all there was to a cornice then all ought to be equally good so the differences between these three and the truly random staircases of figure 6 give some clues as to what the algorithm lacks, but may by chance produce.

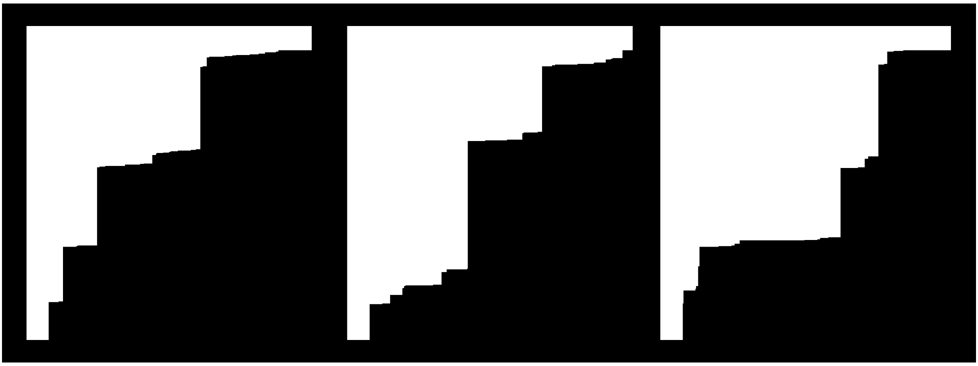

Figure 7. Selected Levy staircases, $D = 0.6$.

Most likely the chosen examples seem superior because they happen to fall into three parts, so conforming to our expectation of how a traditional cornice is ordered, and also because they display that they are scaling more conspicuously than others that are either too jumpy or too smooth. Breaking the cornice into pieces that are roughly one third the size of the original is in accordance with what is suggested by Salingaros (1998), who recommends scaling factors close to three as the best for producing natural-looking fractals. All this suggests that mouldings are scaling forms although there must be principles of proportion and ordering at work that go beyond the scope of the algorithm. Interestingly, Gothic mouldings also seem to be fractal. Figure 8 shows a plan of a Gothic column next to the well-known fractal, the Koch curve. The column (Fletcher, 1975, page 718), has small bulges clustered on larger ones over perhaps three stages (in both the column and the fractal the solid portion resembles the negative space). This suggests that being fractal is important for a moulding and goes deeper than the distinction between gothic and classical.

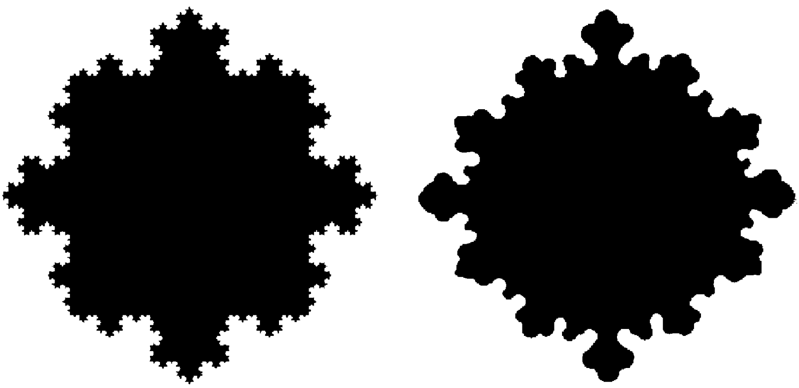

Figure 8. Koch curve and Gothic column compared.

Fractals and picturesque composition

What use is it for a moulding to be fractal? The classical entablature has been interpreted as a representation in stone of a primitive timber structure (Fletcher, 1975, page 205), in which case the theme of three might have emerged as an elaboration of the joining of three pieces of timber. Although it is certainly true that elaborate classical ornament can be assembled out of a surprisingly small number of simple timber components it is, however, barely credible that there can be a practical reason for them being fractal. Mandelbrot (1983, page 375) makes some remarks concerning scaling in music that may be relevant. He points out that when music is being composed it is subdivided into movements characterised by different tempo and loudness, and these movements are further subdivided in the same fashion down to the shortest meaningful subdivisions. The result, he says, is bound to be scaling. This is analogous to the iterative processes that create geometrical fractals. The same reasoning may be applied to architecture, the argument running as follows. There is a process, call it composition, which if applied to the arrangement of form at several different scales will automatically produce a fractal. But what is this thing called composition? It may be humane advice about arranging spaces, as offered by Alexander et al (1977); I have tried to show elsewhere that his approach is likely to lead to scaling forms, particularly around the size range of people (Crompton, 2001). But can architectural composition, in the sense of traditional advice on the arrangement of form, also lead to scaling?

This argument can be reversed—composition may be successful because it produces fractals. I suggest that fractals are attractive and satisfy many of the rules of thumb for good composition that have been observed in painting and architecture. In other words, following rules of composition will tend to produce objects that are scaling; furthermore studying fractals might help explain traditional rules of composition which otherwise seem today to be arbitrary and formalistic. To see why this might be the case we will look at some traditional advice on ornament and composition taken from Ruskin, who has been chosen as an authority because his writings on composition and architecture were the last word on this subject before modern architecture rendered the subject out of date. I have never found anything written on this topic that is not better expressed in Ruskin. Unrau (1978) is a good introduction to his work. This sort of formal approach to design is not taught to architecture students today because it is perceived to be arbitrary and unreasonable, indeed it is felt to be faintly disreputable. This, however, need not be an impediment to this inquiry since the question is not, is it right? but rather, does this approach favour fractal forms?

For Ruskin, composition meant simply putting several things together, to make one thing out of them, (Ruskin, 1904, volume 15, page 161). He likened pictorial composition to composition in music and poetry, and although sceptical of design by rules, remarking that "you might much more easily receive rules to enable you to be witty" (Ruskin, 1904, volume 15, page 163), he is nonetheless not shy of offering advice. What follows is taken mostly from his *Stones of Venice*, *Elements of Drawing*, and *Modern Painters*, works in which his opinions are supported by many apt and uncanny examples of drawings and buildings which obey his rules.

(1) Ruskin believed that an ornament should be designed so that it is meaningful when seen at long, intermediate, and close range. This is well illustrated, for example, by his dislike of the decoration on Constitution Arch in London, where there is a big jump in scale between a patch of intricate cast-iron decoration and the mass of smooth stone in which it is set (Ruskin, 1904, volume 3, page 208; Unrau, 1978, page 90). He wrote "All good ornament and all good architecture are capable of being put into shorthand; that is, each has a perfect system of parts, principal and subordinate, of which, even when the complemental details vanish in distance, the system and anatomy remain visible." In short, ornament ought to reveal new details and forms as one gets closer. Of course a fractal is well adapted to do just that; indeed in their most developed form fractals look the same from whatever distance they are viewed.

(2) Ruskin liked uncountable arrays: "... a sense of power may be communicated ... by a continuous series of any marked features, such as the eye may be unable to number, while yet we feel, from their boldness, decision, and simplicity, that it is indeed their multitude which has embarrassed us, not any confusion or indistinctiveness of form" (1904, volume 8, page 110). An example of what he had in mind here might be

an array of arches, but he includes the lines of mouldings "... on a smaller scale ... of those Greek mouldings, of which, repeated as they now are in all the meanest and most familiar forms of our furniture, it is impossible to altogether to weary" (1904, volume 8, page 110). Again the complexity of fractals supplies the desired effect. Fractals are aptly described as uncountable. If one tries, for example, counting the bumps on the Koch curve one is soon lost because it cannot be decided if a component is too small to be included or not. In the same way the pinnacles on a gothic cathedral evade being counted, and this is a different effect from a large number of windows in a regular array which can be counted methodically, and different again from a random array such as trees in a wood.

(3) Ruskin's *law of principality* advises artists to arrange their material so that one feature shall be more important than all the rest, and that the others shall group with it in subordinate positions (Ruskin, 1904, volume 15, page 164). This traditional rule of composition is often illustrated by a cluster of leaves where one is larger than the others, rather like the fractal patterns produced by iterated function systems. He is insistent that this rule be used subtly, which is most easily achieved if the principal feature is larger than the others by a modest amount and if there are plenty of competing forms. Ruskin's law is satisfied by the largest repeating unit in a fractal. The principal feature may be indicated not only by its size but also by being the focus of a design, which may be achieved if important lines in the design lead to it. Happily this often happens in fractals where groups of similar forms arrange themselves along curves, an example of this is given later. If we try to apply this law to figure 3 then the principal feature would be the longest horizontal edge about a third of the way up the cornice, its right hand end would be the focus of the design since it lies on the curve which in growing from the base of the column envelops the whole entablature.

(4) Another method of expressing unity is to use the *law of repetition*, in which "one group imitates or repeats another, not in the way of balance or symmetry, but subordinately, like a far-away and broken echo of it" (Ruskin, 1904, volume 15, page 167). He illustrates this in action in a drawing by Turner in which the profile of a hill is copied in the shape of a river bank and features are grouped in pairs, a trick I have seen in other paintings by Turner and which is hardly noticeable until pointed out but then seems most strikingly odd. The picturesque device of clustering like elements may be seen in the distribution of horizontal lines in a cornice. Repetition and clustering are the distinctive features of fractals, even those generated by random algorithms. The shapes of clouds where parts resemble the whole exhibit this sort of repetition and are of course fractals. "I have never succeeded in drawing a cumulus", Ruskin reported, "Its divisions of surface are grotesque and endless, as those of a mountain" (Ruskin, 1904, volume 7, page 162).

(5) The *law of continuity*: "... an ... orderly succession to a number of objects more or less similar ... most interesting when it is connected with some gradual change in the aspect or character of the objects." Examples of this sort of continuity might be flanks of valleys or a succession of clouds, in which we see Ruskin's admiration for picturesquely varying series of objects. "If there is no change at all in the shape or size of the objects, there is no continuity; there is only repetition—monotony. It is the change in shape which suggests the idea of their being individually free, and being able to escape, if they liked, from the law that rules them, and yet submitting to it" (Ruskin, 1904, volume 15, pages 170–171). Orderly successions of shapes of changing character and size can be seen in many fractals; indeed, it is the essence of a fractal that it cannot be simply periodic. To be fair it must be pointed out that Ruskin admired, in accordance with this law, arrays which are certainly not fractal, such as diminishing arches in bridges, or arrays which seem regular at first sight but in fact, whether by accident

or design, vary slightly (Unrau, 1978, pages 51 – 64). The law is, however, satisfied by many forms by virtue of their being fractal.

(6) Other advice on composition concerns the organisation of curves, consistency (that is, grouping like with like), and interchange (making two opposing things copy each other), and the use of contrast (Ruskin, 1904, volume 15, pages 180, 191 – 200). Although not obviously concerned with the organisation of shapes that might be scaling, his advice is not inconsistent with producing shapes which are fractals. For example, Ruskin was interested in bounding lines, that is, lines which may be imagined to envelop the silhouette of shapes such as trees, cliffs, and buildings and he is firm in his belief that "... all beautiful objects whatsoever are thus terminated by delicately curved lines" (Ruskin, 1904, volume 15, page 176). By this he meant curves of changing curvature and these are to be found in many fractals, particularly those that grow to resemble vegetable form. Amazingly, he gives a reason why this should be so by drawing a fractal of his own, one that is accurately defined as any in Mandelbrot. It is to be found in *Modern Painters* (Ruskin, 1904, volume 3, figure 56, pages 83 – 84) and is here reproduced as figure 9. His fractal tree is entitled 'Sketch by a clerk of the works', a little joke perhaps which betrays his low opinion of mechanically produced ornament. The rule given for its generation is that each stem separates into two branches at an equal angle, each branch being three quarters the length of the preceding one. Its dimension can now be calculated; it is $\ln 2/\ln 1.33 = 2.43$, a higher dimension than similar trees drawn in Mandelbrot (1983, page 154), who wished to avoid self contact, but Ruskin wanted his branches to overlap like a real tree. Ruskin draws attention to the subtle curve which passes through its extreme points and explains that, although branches in real trees will droop, "... the form in a perfect tree is dependent on the revolution of this sectional profile, so as to produce a mushroom-shaped or cauliflower-shaped mass. This is what renders the contours of tops of trees intensely difficult to draw rightly ..." (Ruskin, 1904, volume 3, pages 83 – 84). This comment illustrates a great theme of Ruskin—"... nothing distinguishes great men from inferior men more than their always, whether in life or in art, *knowing the way*

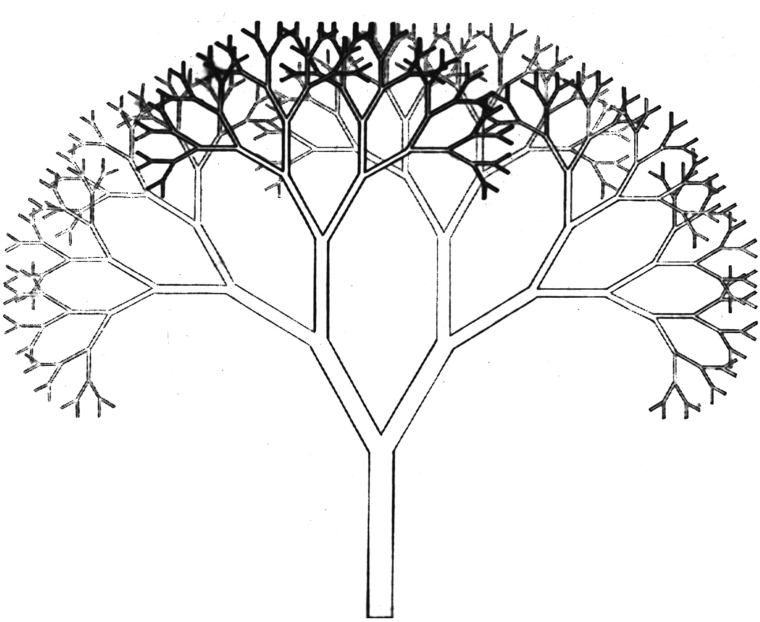

Figure 9. Ruskin's fractal tree—'Sketch by a clerk of the works' (drawn in 1858).

things are going" (Ruskin, 1904, volume 15, page 91). This knowledge is of course gained by studying nature; how pleasing that he understood that the outline of a tree is formed by a process of iteration.

Ruskin's ideas on composition were much influenced by the study of clouds, rocks, trees, and leaves, indeed their morphology seen from an artist's point of view forms a large portion of *Modern Painters*, parts of which are, all told, almost a manual of fractals in nature. A great deal more could be written on this. In Ruskin the sympathetic study of nature and architecture is combined so perhaps it is not surprising that his appreciation of buildings should be coloured by his recognition in them of natural forms that would be today called fractal.

Conclusion

We are now in a position to say what makes some of the randomly produced cornices of figure 6 better than the others; the better ones happen to conform to principles of design that are rooted in a study of nature and picturesque architecture. Ruskin's advice on composition is obeyed by a classical entablature because its complexity is revealed gradually as one gets closer, and its features are impossible to count. It obeys the law of principality insofar as it has a largest step surrounded by smaller copies clustered in threes according to the law of repetition, and its repeated steps are organised according to the law of continuity in a slightly disordered series. It obeys these rules by virtue of being a fractal. The same can be said of the Koch curve and Gothic moulding in figure 8, and of many of the fractals in Mandelbrot's book.

Traditional composition will favour fractal forms. If fractals are ubiquitous in nature then perhaps their use offers a new way to copy nature and their use in architecture belongs with other attempts to introduce natural forms into design, an idea which interested Ruskin and so many others. Not only are we possibly predisposed to recognise and like them, but the opposite, a lack of scaling features, may feel peculiar. Salingaros and West (1999) who cite examples of the hyperbolic distribution of sizes leading to scaling forms in social as well as in the natural sciences, go so far as to suggest that departures from scaling forms result in alien incoherent structures. Because using fractals can lead to forms that satisfy traditional criteria for well-composed architecture, it may be that the lack of interest in formal composition in modern architecture is one of the causes of its lack of scaling features.

References

Alexander C, Ishikawa S, Silverstein M, 1977 *A Pattern Language* (Oxford University Press, New York) pp 311 – 314

Crompton A, 2001, "The fractal nature of the everyday environment" *Environment and Planning B: Planning and Design* **28** 243 – 254

Fletcher B, 1975 *A History of Architecture* 18th edition, revised by J Palmer (Athlone Press, London) page 718

Mandelbrot B, 1983 *The Fractal Geometry of Nature* (W H Freeman, San Francisco, CA)

Normand C, 1931 *A Parallel of Orders of Architecture* reedited by R A Cordingley (John Tiranti and Company, London)

Ruskin J, 1904 *The Works of John Ruskin* 39 volumes, Eds E T Cook, A Wedderburn (George Allen, London)

Salingaros N A, 1998, "A scientific basis for creating architectural form" *Journal of Architectural and Planning Research* **15**(4) 284 – 285

Salingaros N A, West B J, 1999, "A universal rule for the distribution of sizes" *Environment and Planning B: Planning and Design* **26** 909 – 923

Unrau J, 1978 *Looking at Architecture with Ruskin* (Thames and Hudson, London)