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A statistical examination of visual depth in building elevations

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Abstract. Old buildings may be easier to read than modern buildings because they possess visual depth. This is the finding of an experiment that analysed building elevations and newspapers looking for a connection between their character and the size – frequency distribution of their component parts. It was found that drawings of older buildings sometimes displayed $1/f$ scaling and thus resembled natural scenes, but that this was rarely the case for modern buildings. The same distribution was also found in newspaper pages. We hypothesise that this distribution allows buildings to appear interesting and changeful when approached from afar in the same way that it makes newspapers easy to read over a range of distances, on a newsstand, over a shoulder, and so forth.

Introduction

Our experiment aimed to discover if elevations of buildings in different styles could be distinguished statistically. In order to do this, drawings of buildings were treated as assemblies of parts whose areas were measured without any regard for their shape or arrangement. As will be seen, this severe simplification still allowed us to say something interesting about buildings, even though it eliminated most of the orderly qualities we associate with architecture. This simplification was based on what is known as the 'dead leaves' model, a method of analysing images by treating them as tessellations of uniform patches whose variations of size and shade can then be studied statistically. It has been found that photographs from databases of natural images examined this way often show scale invariance, something that has also been found to occur with images made by randomly arranging simple shapes in layers (Lee et al, 2001). The significance of this is that, even if images of natural scenes are statistically similar to images of random shapes in space, the images themselves are not random. Knowing that an image was scale invariant, a gambler could make a better than random guess as to the size of unseen patches in an image. Indeed it seems that nature takes advantage of these subtle regularities because biological vision researchers have been able to explain visual processing strategies by combining results like this with optimisation principles (Ruderman, 1997).

In a recent example of this sort of approach that is relevant to built-environment research, Yang and Purves (2003) scanned the distances to all the objects in the field of view in campus and woodland scenes and found that the probability that an object is a particular distance from the observer follows a remarkably uniform pattern: the distribution of distances to objects was scaling, with a maximum probability occurring close to three metres. They then ingeniously used this result to explain no less than six illusions of distance perception experienced in simplified laboratory environments. Their theory is that the visual system has evolved to make the best statistical guess about distances based on past experience. It is when this expectation is upset that we experience minor illusions of distance. In short, we expect the world to be scaling and may be deceived when it is not.

Studies like these tell us that, not only are we sensitive to the size–probability distribution of objects around us, but also that we expect the distribution to take a certain form. In particular, we seem to be predisposed to expect the world to be scaling. Will this have implications for how we perceive the built environment? To answer this we need to know what the size–probability distributions are for buildings when viewed as a collection of parts along the lines of the dead leaves model. Will they show the same scaling distribution as pictures of trees and rocks? If the parts of a building are laid out in order of size will they diminish in a smooth sequence or will there be missing scales or repetition? In other words, just how do buildings go to nothing? It was with these questions in mind that our tests were performed.

Method

The nine buildings used in our experiment are listed below and are shown, all to different scales, in subsequent figures (figures 4, 5, and 6). Drawings of them were taken from Weston's (2004) collection of famous buildings, except for those marked with an asterisk, which were specially surveyed for this study. Two broadsheet newspapers were analysed: *The Daily Telegraph*, 2006, and *The Morning Post*, 1890—the blocks into which they were divided are shown in figure 5. *The Morning Post* (1772–1937) ceased publication when it was acquired by *The Daily Telegraph*, and may be regarded as an old version of that paper; its front page shows advertisements, not news, and the largest blocks on its grid, marked with a dot, are for births, marriages, and deaths. To these were added, by way of a control, patterns scanned from Le Corbusier's book *The Modulor* (1954, page 95) and a camouflage pattern resembling foliage from Moriizumi's collection (2002, patterns 060 and 061).

(1) Eames House 1949, C. & R. Eames.
(2) Computer Building * 1970, BDP, Manchester University.
(3) Stockholm Public Library 1927, E G Asplund.
(4) A set of patterns from *The Modulor* * (Le Corbusier, 1954).
(5) Schröder House 1924, G Rietveld.
(6) Venturi House 1964, R Venturi.
(7) Durnford School * 1908, E Wood & J H Sellers, Lancashire.
(8) Semidetached house,* Manchester, 1936, designer unknown.
(9) Unity Temple, 1908, F L Wright.
(10) Camouflage pattern (Morrizumi, 2002).
(11) *The Morning Post*,* 12 December 1890.
(12) *The Daily Telegraph*,* 30 January 2006.
(13) Deaf and Dumb Institute,* 1878, Manchester (see figure 1).

The method of analysis was to trace the outline of every identifiable element on each elevation using VectorWorks (Nemetschek, Columbia, MD), a program that was also able to calculate and list the areas of elements. The process could not be made automatic because judgment was needed to decide what counted as an element, the difficulty coming from having to decide whether to count gestalts along with their components. The rule was followed that combinations of parts were to be counted along with the parts themselves. For example, in figure 1 windows occur in pairs and were counted both individually and as a double. The inclusion of gestalts is a slight departure from the 'dead leaves' model, in this experiment every nameable part of a building was counted. The area of the whole building was taken as one unit and its parts were measured as a ratio of that quantity, typically their values ranged over three to four orders of magnitude.

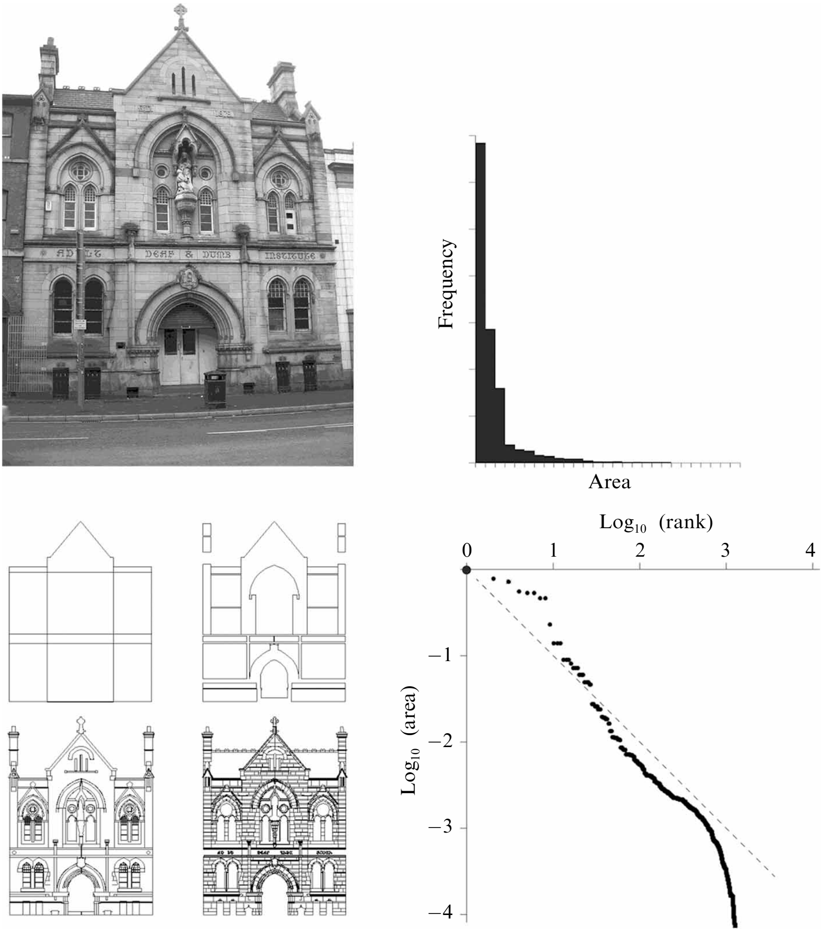

Figure 1. Deaf and Dumb Institute, histogram and Zipf plot compared.

Visualising size – rank distributions

The problems of visualising the data are illustrated in figure 1, which shows the Deaf and Dumb Institute, a Manchester municipal Gothic building. The drawing created by the tracing process is shown, separated into four parts for clarity. The areas of its 1302 components were found to range over more than four orders of magnitude from the largest part, the main gable, to the smallest, which was part of a moulding. The distribution of their sizes is shown in the histogram in figure 1. Notice that there is no sense of an average-sized component, most components are small and the distribution is shaped something like a hyperbola with a long tail. In fact the tail continues well outside the figure, but the black bars are not tall enough to be seen clearly for the more infrequent larger parts of the building. Histograms for all the examples in this study would look something like this and are generally rather uninformative, because if the vertical scale is suitable for the most common elements the less common ones hardly register.

An improvement on the histogram comes if we change from linear to logarithmic scales. This compresses the wide range of data to a manageable size, and since as the Weber – Fechner law tells us, we respond to the logarithm of a stimulus, such scales correspond better to our perception. Such a graph is shown in the bottom right of figure 1, in it each point represents one of the components, although they are often too close together to be distinguished. The parts of the building are ranked in order of area with the largest ranked number one and given an area of one unit. Because $\log_{10}(1) = 0$, this point occurs at the origin of each plot and represents the whole building considered as a single object. The other components are plotted according to their values of $\log_{10}$ (rank) and $\log_{10}$(area) and fall away as they get smaller and smaller. This is a Zipf plot, and the shape of its string of points can be interpreted to give information about just how the components diminish to nothing. This is done by reading the string of points as approximating straight-line segments of varying gradients, as will now be shown.

In figure 2, sets of twenty-five squares diminishing at different rates are compared. All of them have straight-line Zipf plots; a shallow gradient indicates that squares diminish slowly in size, a steep line that they diminish quickly. Their areas, a, were generated by the formula:

$$a(x) = a(1)x^D, \qquad x = 1, 2, 3, \ldots, 25 \ , \tag{1}$$

using different values of D as noted.

Each set is displayed in four different ways:

(1) as a $\log_{10}$ (rank) – $\log_{10}$ (area) Zipf plot;

(2) as a nested set;

(3) as a cluster of squares randomly arranged (like dead leaves);

(4) set out in order of height.

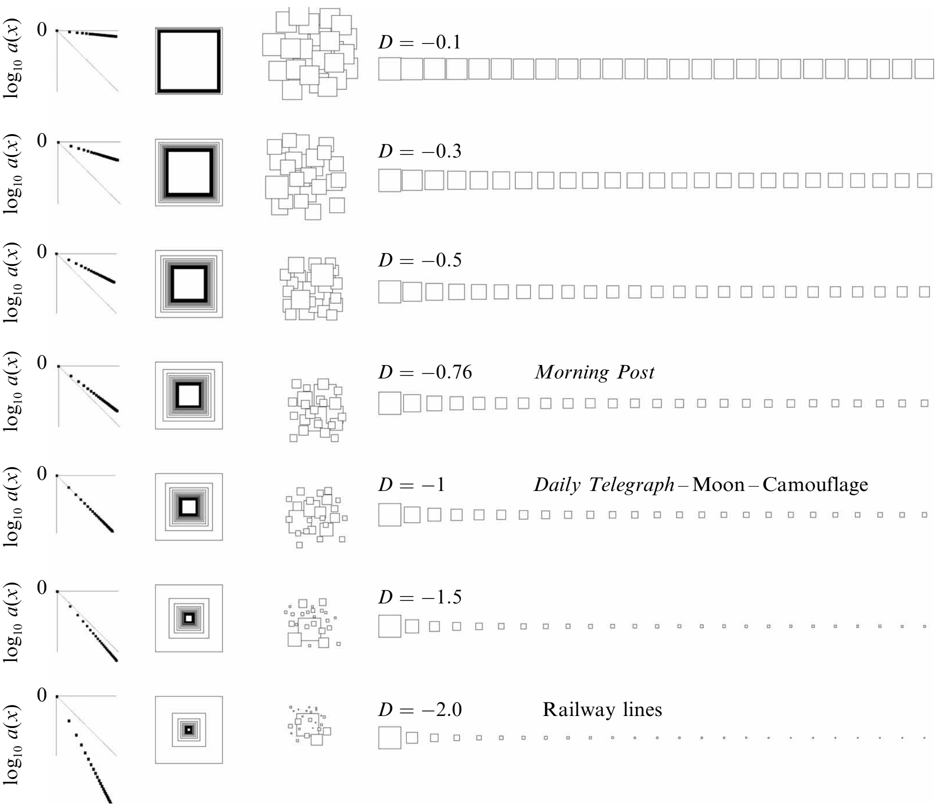

Figure 2. Examples of scaling series of squares diminishing at different rates.

The formula [equation (1)] creates series that are scaling. What this means is this: suppose that all the squares are enlarged by a constant factor k^D, then, because $a(kx)/a(x) = k^D$, the squares numbered x become the same size as the squares previously numbered kx. If the series of squares is long enough for us to overlook its biggest and smallest members then enlarging the set may seem to have no effect. To this extent these sets of squares seem to be scale free and to repeat themselves inwardly—they may be considered a sort of fractal.

From equation (1) it follows that:

$$\mathrm{P}(A > a) = \text{constant} \times a^{1/D} , \tag{2}$$

where $\mathrm{P}(A > a)$ is the probability that an area A chosen randomly from the set is bigger than a. This is the definition of a hyperbolic distribution (Mandelbrot, 1983, page 341), which, when D is close to -1, is also known as a $1/f$ distribution. The fifth series in figure 2 is of this type exactly; with a gradient of -1 the areas of the squares so generated are in the harmonic series 1, 1/2, 1/3, 1/4, 1/5 ... . This distribution—which happens to be, for example, the distribution of areas of moon craters—is often found in nature (Mandelbrot, 1983, pages 302, 305). Other examples can be found in Voss (1989).

At the bottom of figure 2 we see a set in which scaling occurs in a particularly clear way. When $D = -2$ the squares represent the sizes of equally spaced equal objects seen from a point, such as railway sleepers viewed down the track. If scaling is taken to mean, roughly speaking, that things stay the same when enlarged, then we can see scaling in action when we walk along a railway track, the sleepers keep getting larger to the eye whilst the view stays the same.

In contrast to these examples, two Zipf plots for nonscaling distributions are shown in figure 3. Figure 3(a) shows the values of a geometrical series: r^0, r^1, r^2, r^3,... with $r = 1.1$; unlike the straight lines in figure 2 this Zipf plot dives away smoothly because it lacks small values—to be scaling, a set of squares needs more small sizes than a geometrical series can provide. (It is worth noting that a set of objects in geometrical progression is not scaling, even though resizing them by a factor r leaves them unchanged. The constant ratio of adjacent members of geometrical series is a special case of scaling, but because r^{2n}/r^n does not equal r^2/r, the series cannot be made to lie over itself when rescaled.) Figure 3(b) shows a typical Zipf plot for twenty-five random numbers between 0 and 1. It meanders its way from large to small without any pattern. Although it appears to be governed by a curve this is only caused by the

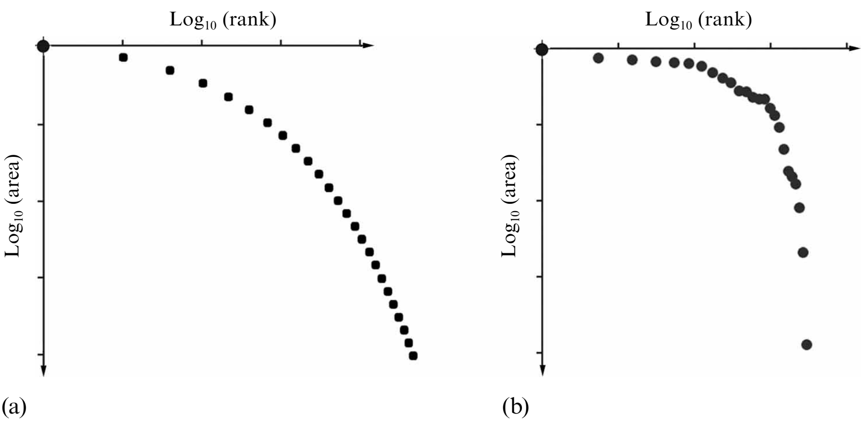

Figure 3. Zipf plots for nonscaling series of numbers: (a) numbers in geometrical series; (b) twenty-five random numbers.

logarithmic scale compressing the values to the right. This is the sort of Zipf plot expected if there is no relationship between the areas of the set of shapes. Bearing these examples in mind we can now examine the Zipf plots for the buildings and newspapers shown in figures 4, 5, 6, and 7. Each point in these figures represents an object—where large objects are on the left and the tail on the right represents a cut-off, at which objects become too small to be counted. Ignoring these tails, the Zipf plots fall into three types: (1) stepped; (2) not straight; (3) straight. These may be correlated with types of architecture.

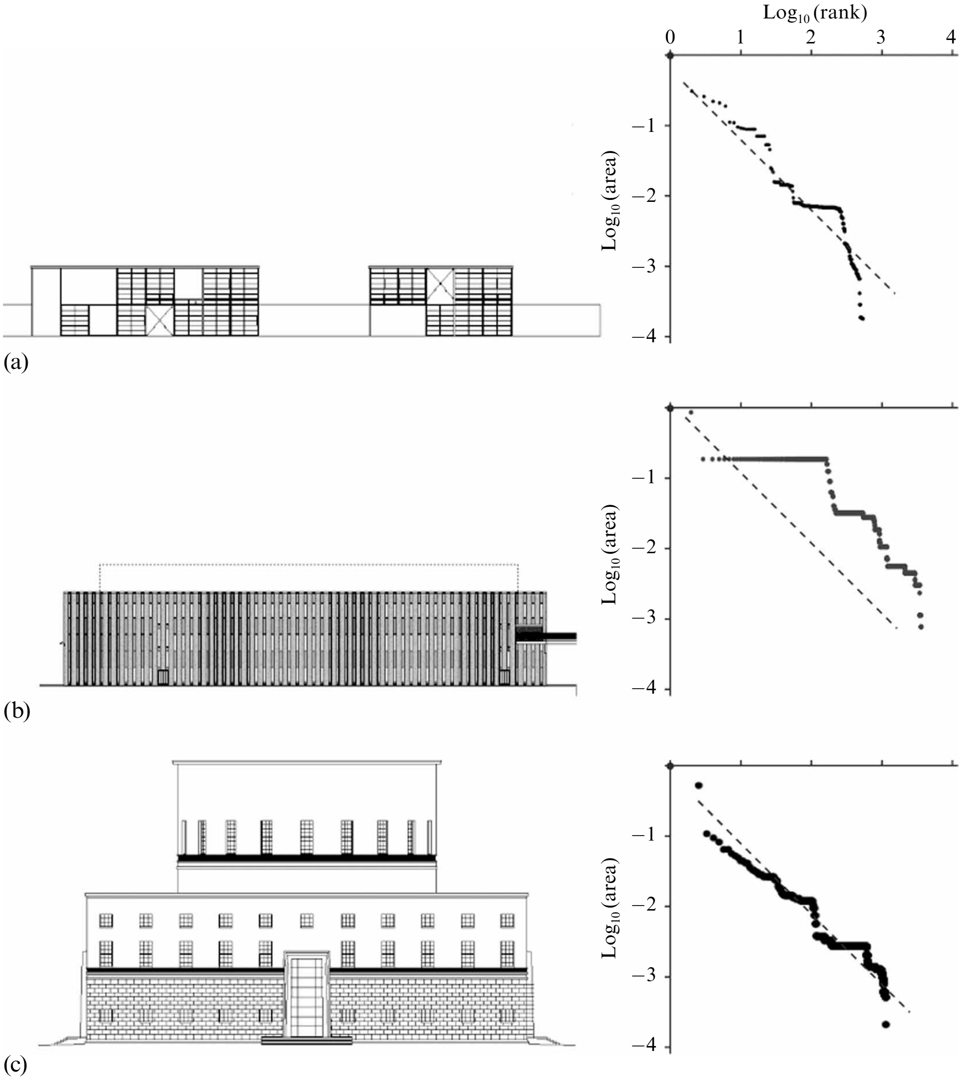

Figure 4. Zipf plot is a stepped line: (a) Eames House, (b) Computer Building, (c) Stockholm Library.

Buildings for which the Zipf plot is a stepped line

Because a horizontal line in a Zipf plot represents objects of uniform size, these stepped plots are the sign of a repetitive building. Both the Stockholm Library and the Eames House show this pattern to some degree, but the most exaggerated in this respect is the Manchester Computer Building designed by BDP in 1970 (figure 4).

Richard Saxon, chairman of that firm, told one of us that its resemblance to an IBM punch card was not accidental. The first level step on its Zipf plot represents the uniform brick piers, other steps represent the equally repetitive window and parapet stones. No less significant are the steep parts of the line that indicate a lack of components over that range of sizes. This means that there is no chain of intermediate components connecting the small parts and large parts. Note also that in this rather extreme example there are no components intermediate in size between the vertical strips and the whole building. Something similar occurs in the Stockholm Library, where there is little at the scale between the window, blocks, and the width of the building. These buildings show a lack of hierarchy of sizes, something that has been criticised as unnatural in architecture, notably by Salingaros (2000).

Buildings for which the Zipf plot is an irregular curve

The two examples in figures 5(a) and 5(b) show curving Zipf plots. The modulor arrays copied from Le Corbusier's book are drawing exercises showing how a square may be divided using the proportions of 0.612 and 0.5. The method of division leads automatically to a few preferred dimensions being repeated and makes the Zipf plot go in steps. The proportioning system causes many of these dimensions to be in a geometrical progression, explaining why, aside from its steps, this plot resembles the curve for the geometrical series in figure 3. A similar proportional division of the facade of the Schröder House makes a smoother curve. On the other hand, the Venturi House Zipf plot shows little pattern, in fact one could say that it does not differ much from the Zipf plot for random numbers in figure 3. It certainly lacks detail at small scales compared with the other examples. These two latter buildings have little decoration and, even if small components may be related to the larger ones in a proportional system, there is a paucity of small items. A curving Zipf plot may be regarded as the signature of a building designed on a system of repeating proportions.

Buildings for which the Zipf plot is a straight line

The examples in figure 6, together with the Deaf and Dumb Institute in figure 1, all show good straight lines with gradients close to -1. They are all well-composed buildings and, in the case of the Durnford School, even a little picturesque. The areas of their component parts follow a $1/f$ distribution and in this regard they resemble photographs of natural scenes. All of them have a lot of small-scale detail that is related to the larger components by a chain of intermediate sizes. They also resemble parts of themselves—that is, parts of these buildings can be seen to copy other parts at a different scale—for example, in the Unity Temple the right-hand is half is a simplified version of the left-hand half, in the Deaf and Dumb Institute an arched opening occurs over a range of sizes. This sort of inner repetition does not seem to be used in the nonscaling examples, such as the Venturi or Schröder houses.

The same distribution of sizes was found in the newspapers we examined; although they are different every day, the sizes of stories within the grid of seven or eight columns shows a surprising statistical regularity. The areas of 285 blocks that make up the *Post* and the 64 blocks that make up the *Telegraph* show good straight lines, with gradients close to -1, showing that their areas follow a $1/f$ distribution. The papers are so regular that, provided we set sensible limits for maximum and minimum sizes, we can derive a formula for the probability that an article exceed a certain size a, enabling us to predict the likelihood of big stories.

For example, in the case of the *Telegraph*:

$$\mathrm{P}(A > a) = \text{constant} \times a^{-1} \ . \qquad (3)$$

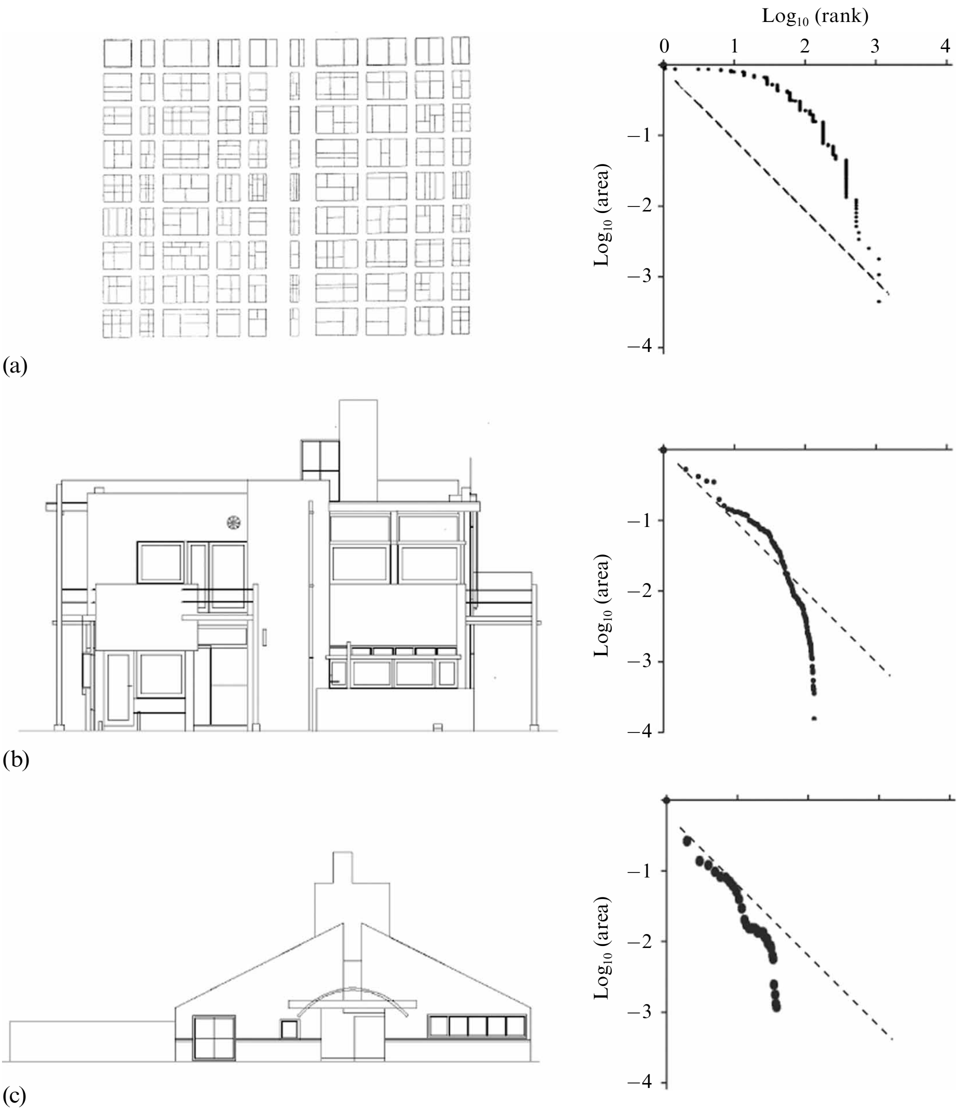

Figure 5. Zipf plot is an irregular curve: (a) Modulor Array, (b) Schröder House, (c) Venturi House.

In laying out a newspaper page a compositor will be mindful of factors such as balance, movement, and proportion, and will make judgments about the relative importance of elements. This process of composition is a traditional design approach and is possibly the reason why newspapers, statistically at least, resemble older buildings more than modern buildings. Reasons why picturesque design methds are likely to produce scaling forms may be found in Crompton (2002).

The camouflage pattern also shows a beautiful straight line. Since it is designed to appear natural this might not be surprising, but many of the patterns in Moriizumi's book, from which it was taken, are certainly not scaling. This example was chosen because it looked like foliage and possibly it has obtained the quality of scaling as a by-product of this resemblance. Scaling is a useful property for a camouflage pattern to have because in resembling parts of itself it is difficult to be sure how far you are away

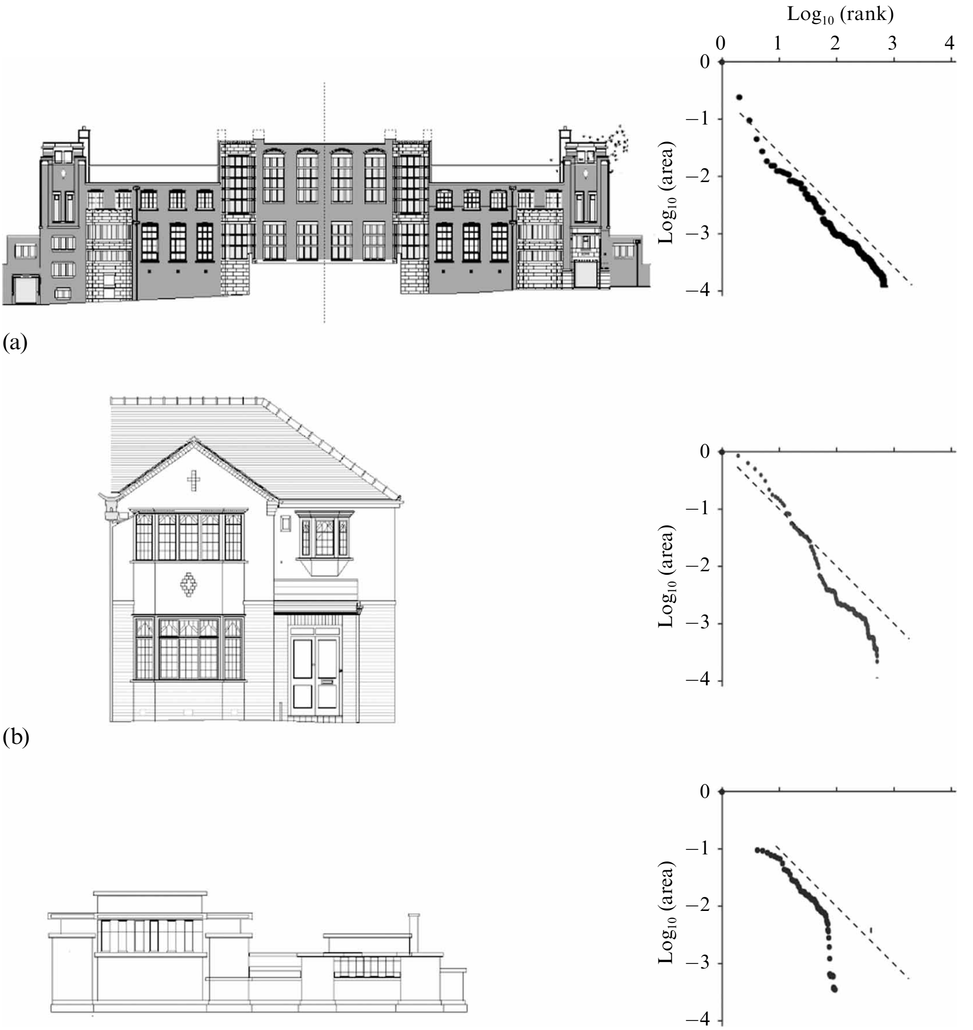

Figure 6. Zipf plot is for the most part a straight line: (a) Durnford School, (b) Semidetached house, (c) Unity Temple.

from it. The same distribution of sizes has been found in the squiggle paintings of Jackson Pollock, in a study that linked these seemingly chaotic works with nature (Taylor et al, 1999).

Discussion

Visual depth, the degree to which an environment is able to keep revealing fresh features as it is approached, is something that rank – size Zipf plots allows us to describe and quantify. In general, visual depth seems interesting and natural. Visually deep buildings exhibit detail over a complete range of sizes, and therefore present new information to a viewer in a continuous stream as one walks towards them; conversely, buildings lacking parts in a particular size range or lacking small-scale detail may appear blank or dull. This is as important in virtual environments as it is in the real world, where a lack of visual depth in objects is one of the ways that they appear artificial. These issues are not new—John Ruskin wrote about them at length;

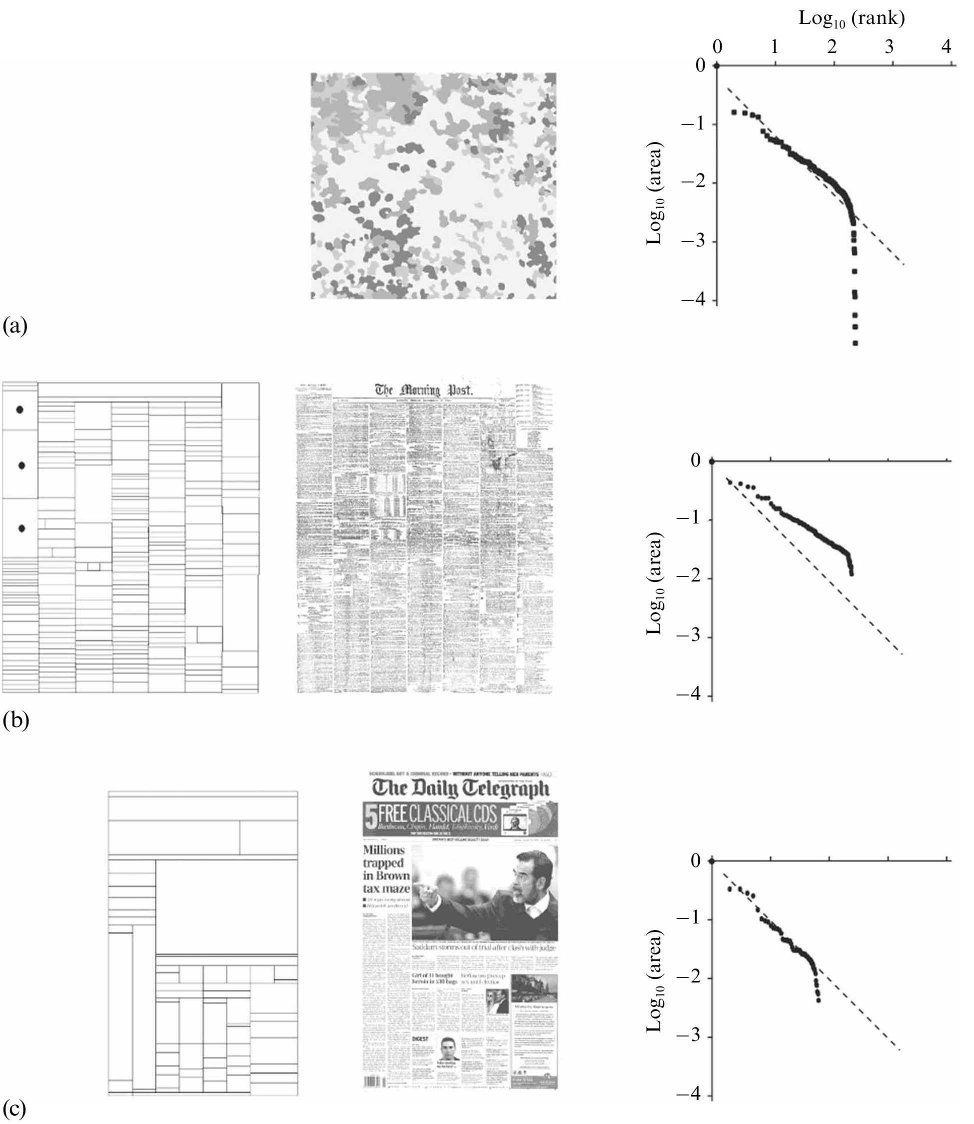

Figure 7. Zipf plot is for the most part a straight line: (a) Camouflage patch, (b) *The Morning Post* newspaper, (c) *The Daily Telegraph* newspaper.

he was fond of drawing buildings over a range of distances to see how they changed as details emerged from the shadows (Ruskin, 1904, volume 15, figure 22). So fundamental was this dynamic view of architecture to him that, in a survey of Ruskin's writings on architecture, Unrau (1978) found it convenient to divide the writer's opinions into chapters on ornament viewed at long distance, intermediate distances, and at close range. A modern version of this argument is to be found in Salingaros (2000), where the author reasons that buildings ought to possess features in a scaling hierarchy of sizes, justifying this by likening buildings to complex hierarchical systems found in nature and engineering. Other reasons why hierarchies of sizes are most naturally based on power law, such as equation (1) here, may be found in Salingaros and West (1999). It is worth noting that it seems to be difficult to produce a scaling building using a modernist vocabulary. The Schröder House, for example, has been beautifully designed down to the smallest detail of windows and fittings, and yet it still lacks detail at a small scale even when compared with something as prosaic as the 1936 Semidetached house.

What differences can be observed between the buildings in figures 1, 6, and 7 that display scaling and those that do not? Putting the two sets side by side one might perhaps say that the scaling buildings are richer in detail and that, where they are repetitive, the repetition has been tempered with some variation. It is difficult to be sure of course, because the drawings are so small, but this is not an irrelevant difficulty, because at the heart of this problem is the question: what is the connection between big and small? This is something that is difficult to represent in a drawing because drawings always stop at some level of detail: only the simplest buildings can be absorbed in a single glance. In fact, the difference between scaling and nonscaling buildings is something that is seen dynamically as they are approached from a distance. Nonscaling buildings like the Computer Building [figure 4(b)] show themselves all at once or in bursts of same-sized components. Scaling buildings reveal themselves continuously and gracefully, they cannot of course possess infinite depth like railway lines—that would be a physical impossibility—but they can smoothly present themselves at all scales down to a small unit, such as a brick or a piece of a moulding. They can engage our interest at all distances and can be read from afar as well as nearby. In possessing this quality of readability scaling buildings resemble newspapers.

Conclusion

Buildings and newspapers can be surprisingly similar. Both *The Daily Telegraph* and the Deaf and Dumb Institute are laid out in columns with a title, date, and a human image, a photograph in the first case and a statue in the second. Moreover, grids of newspapers, when turned sideways, resemble lightly irregular fenestration patterns currently fashionable with architects such as MVRDV, Erick van Egeraat, and others. Figure 8 compares a newspaper grid with a Manchester housing block by Andrew Wallace Architects built in 2006. The fact that buildings and papers may both have a $1/f$ distribution of sizes makes the resemblance stronger. These architects seem to be deliberately making varied and visually deep buildings. Scaling seems to have occurred naturally in the past through the use of ornament and composition, but if their use is not acceptable to architects today it may still perhaps be possible to introduce scaling by devices such as the irregular grid seen in figure 8, which, from this point of view, may be seen as an example of a sort of picturesque modernism. Who knows what other

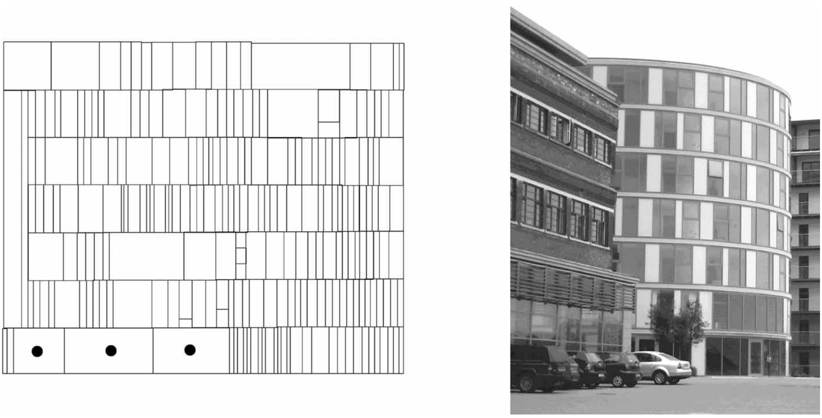

Figure 8. Newspaper grid and a building elevation compared.

possibilities exist for introducing scaling into a modern vocabulary of design until we look for them in an organised way?

A final thought: in newspapers the scaling manifests itself in the repetition of the familiar motif of picture–heading–article over a range of sizes. Of course this also makes newspapers easy to read over a range of distances, on a newsstand, for example, or across a table. Since they are products evolved in a competitive market this is doubtless deliberate. Odd though it may seem, this study of buildings has accidentally led to an explanation of something which has hitherto not been the subject of a serious scientific enquiry: why newspapers are easy to read over someone's shoulder.

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